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**A Study of Liouville Green Transformation for Solving Differential-
Difference Equations**

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ABSTRACT

The Liouville-Green transformation, also known as the WKB (Wentzel–Kramers–Brillouin) method, is a powerful asymptotic technique traditionally used for solving linear second-order differential equations with varying coefficients. Its application has recently been extended to differential-difference equations (DDEqs), which involve both derivatives and discrete time shifts or delays. These equations often arise in physics, engineering, and biological systems, where the current state of a system depends on its past behavior and rate of change. The Liouville-Green transformation offers a systematic approach to approximate solutions of DDEqs by converting the complex mixed-type equations into more manageable forms using asymptotic expansions. The method involves transforming the original equation into a second-order linear differential equation without delay, where the dominant behavior of the solution can be studied in terms of rapidly varying or slowly varying functions. By applying this transformation, one can derive approximate solutions that are particularly useful for equations with small parameters or slowly varying coefficients. This paper explores the adaptation of the Liouville-Green method for solving linear DDEqs, outlines the transformation process, and presents example problems demonstrating its applicability. The results confirm that this technique provides accurate approximations, especially in boundary-layer problems and systems with slowly varying dynamics and small delays.